Solutions to Homework 6

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Please report any typos or mistakes to me.

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1. (pg 111, 4) $f(x_1) = f(x_2) \implies (x_1, y_0) = (x_2, y_0) \implies x_1 = x_2$, hence f is injective. $f(X) = X \times \{y_0\}$. Let $U \times V$ be a basis element of $X \times Y$ then $f^{-1}(U \times V) = U$ if $y_0 \in V$ and $= \phi$ if $y_0 \notin V$, both of which are open. Hence f is continuous. Define $h : X \times \{y_0\} \to X$ by $h(x, y_0) = x$. Its easy to check that f and h are inverses of each other and hence both are bijective onto their images. Let U be open in X, $h^{-1}(U) = U \times \{y_0\} = (U \times Y) \cap (X \times \{y_0\})$, hence open in $(X \times \{y_0\})$. Hence h is continuous. Hence f is an imbedding. Similarly g is an imbedding.

2. (pg 111, 5) Define $f : (a, b) \to (0, 1)$ be by $f(x) = (x - a)/(b - a)$. You can check that f takes (a, b) to $(0, 1)$ and f is continuous as its a restriction of a linear map. Define $g : (0, 1) \to (a, b)$ by $g(x) = a + (b - a)x$. g is continuous as its a restriction of a linear map. You can be checked that f and g are inverses of each other. Hence f (and g) is an homeomorphism. Note that the same functions work for the closed intervals.

3. (pg 111, 10) Let $U \times V$ be a basis element of $B \times D$. Then $(f \times g)^{-1}(U \times V) = f^{-1}(U) \times g^{-1}(V)$. Since f and g are continuous, $f^{-1}(U)$ and $g^{-1}(V)$ are open in A and C respectively. Hence $f^{-1}(U) \times g^{-1}(V)$ is open in $A \times C$. Hence $(f \times g)$ is continuous.

4. (pg 111, 11) Fix $y_0 \in Y$, define $f : X \to X \times Y$ by $f(x) = (x, y_0)$. We have seen in Problem 4 above that f is continuous. $h(x) = F(x, y_0) = F(f(x)) = F \circ f(x)$. Since F and f are both continuous, their composition is continuous and hence h is continuous. Since y_0 was chose arbitrarily, this is true for every $y_0 \in Y$. Hence F is continuous in the first variable. Similarly $k(y) = F \circ g(y)$, for the function g from Problem 4 and hence continuous. Hence F is continuous in each variable.

5. (pg 111, 2) Not true. Let $f : \mathbb{R} \to \mathbb{R}$ be the constant function $f(x) = 0$. Then the 1 is a limit point of $(0, 1)$ but the image of f which is the singleton set $\{0\}$ has no limit points.