Solutions to Homework 2

Topology MA 434/542 Fall 2006
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Please report any typos or mistakes to me.

Problems on pg 20 are straightforward and set theoretic. Please ask me if you have questions on those.

1. (pg 44, 6a) Define $f : \{0,1\}^n \to \mathcal{P}(A)$ by $f(a_1, \ldots, a_n) = B$ where $i \in B$ if $a_i = 1$ and $i \notin B$ if $a_i = 0$. Define $g : \mathcal{P}(A) \to \{0,1\}^n$ by $g(B) = (a_1, \ldots, a_n)$ where $a_i = 1$ if $i \in B$ and $a_i = 0$ if $a_i \notin B$. It is easy to check that f and g are inverses of each other and hence bijections.

2. (pg 44, 6b) $|\{0,1\}^n| = 2^n$ and hence $|\mathcal{P}(A)| = 2^n$. Hence if A is finite then $\mathcal{P}(A)$ is finite.

3. (pg 51, 4a) Let $\overline{\mathbb{Q}}$ denote the set of all algebraic numbers and let $\mathcal{P}_n$ denote the set of all polynomials of degree n. For any $p \in \mathcal{P}_n$, let $R_p = \{\alpha \mid p(\alpha) = 0\}$. As a degree n polynomial has finitely many roots, this implies R_p is finite for all p. $\mathbb{Q}^{n+1}$ is countable as it is a finite product of countable sets. Since $\mathcal{P}_n \leftrightarrow \mathbb{Q}^{n+1}$, $\mathcal{P}_n$ is countable. $\bigcup_{p \in \mathcal{P}_n} R_p$ is a countable union of finite (and hence countable) sets and hence countable. $\overline{\mathbb{Q}} = \bigcup_{n \in \mathbb{Z}_+} \left(\bigcup_{p \in \mathcal{P}_n} R_p \right)$ is a countable union of countable sets and hence countable.